32. In a discussion of the Peaucellier[1] Cell by analytic methods the following equations are obtained:

$$(1)\ (x_2 - x_1)^2 + (y_2 - y_1)^2 - b^2 = 0; \qquad (2)\ (x_3 - x_1)^2 + (y_3 - y_1)^2 - b^2 = 0;$$

$$(3)\ (x_2 - X)^2 + (y_2 - Y)^2 - b^2 = 0; \qquad (4)\ (x_3 - X)^2 + (y_3 - Y)^2 - b^2 = 0;$$

$$(5)\quad x_2^2 + y_2^2 - K^2 = 0; \qquad (6)\quad x_3^2 + y_3^2 - K^2 = 0;$$

$$(7)\ x_1^2 + y_1^2 - 2cx_1 = 0.$$

The result of eliminating x_1, y_1, x_2, y_2, x_3, y_3 gives an equation of the first degree, which establishes that the linkage will trace a straight line. There are various ways of effecting this elimination.

1. What element of the situation is left unused by the following procedure in the elimination?

(*a*) From equations (1), (3), (5) eliminate x_2 and y_2 and obtain an equation

$$(8)\ f_1(x_1, y_1) = 0.$$

(*b*) From equations (2), (4), (6) eliminate x_3 and y_3 and obtain an equation

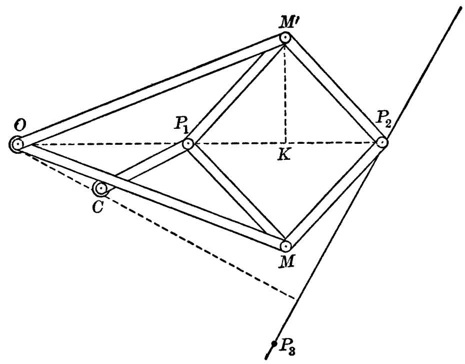
$$(9)\ f_2(x_1, y_1) = 0.$$

(*c*) From equations (7), (8), (9) eliminate x_1 and y_1 and obtain the desired equation.

2. How should this procedure be supplemented to secure the result?

Reply by G. H. Ling, University of Saskatchewan.

The configuration discussed has a line of symmetry; the equations developed show that the points (x_2, y_2) and (x_3, y_3) are *either coincident or symmetrical with respect to the line of symmetry*, but they fail to express that the essential feature of the configuration is that the two points mentioned are *not* coincident.



The suggested procedure for elimination cannot take account of this unexpressed fact. Equations (8) and (9) are identical and the elimination of x_1 and y_1 from (7), (8) and (9) halts because of the lack of three independent equations.

The procedure may be supplemented as follows:

If two of the three equations (1), (3), (5) be solved for x_2 and y_2 two distinct solutions are obtained and these must be the values for x_2 and y_2, and for x_3 and y_3. Both of these sets of values must satisfy that one of the equations (1), (3), (5) which is not employed in the solution just mentioned. The substitution of both of these sets of values in this third equation yields two equations (8) and (9) which associated with the equation (7) yield a single resultant equation free from x_1 and y_1. This last equation is the equation of the desired locus. (See May Monthly, p. 188.)

It is easy to see that the case here discussed can be generalized, though in the more general cases the treatment of the problem would not be nearly so simple.

DISCUSSIONS.

Every teacher of trigonometry, analytic geometry, and the calculus has at times experienced the difficulty of inducing students to regard the radian measure of an angle as a pure number. How is this difficulty to be met? Professor Car-

[1] If reference is made to the article on "Linkages" in the December, 1915, Monthly, by Mr. Leavens, the following coördinates may be applied to his figure (reproduced above): O (0, 0); C (c, 0); P_1 (x_1, y_1); $M(x_2, y_2)$; M' (x_3, y_3); P_2 (X, Y).

ver's discussion below, covering substantially the ground of a paper presented by him at the summer meeting of the Mathematical Association of America in September, 1918, should be suggestive and stimulating, and will, it is hoped, give rise to further expressions of opinion.

Trigonometric Functions—of What?[1]

By W. B. Carver, Cornell University.

The first idea which our students get of a trigonometric function—say sin x —is that the argument x is an *angle*, a geometric entity. According to this conception, the sine of a right angle is 1 whether one thinks of it as an angle of 90° or of $\pi/2$ radians; while sin 2 has different values according as the 2 means 2 radians, 2 degrees, or 2 right angles.[2] Later the student needs the notion of the number sin x as a function of the *number* x, a functional relation which is not dependent upon any sort of geometric ideas or units of geometric measurement. This second point of view is needed not only by those who specialize in mathematics, but also by the large class of students who go no further than a first course in the calculus, and whose purpose may be entirely utilitarian.

It is the writer's conviction that, certainly in our text-books, and possibly in our teaching, we are not doing as much as we might to help the student across from the one point of view to the other.

In our courses in trigonometry the first point of view must prevail: but the way may be prepared for the second by insisting upon familiarity with the circular measurement of angles. The radian unit should be introduced early and *used frequently* throughout the course. The tables of functions should have a column giving the angle in radians adjacent to the column reading degrees and minutes.[3] In the problems for solution in both right and oblique triangles, the given angles should, in at least a few cases, be expressed in radians. The relation of the number π to this method of measuring angles should be made clear. That there is confusion on this point is indicated by such questions as "Why does π mean 3.1416 in one place, and in another place 180°?"

Analytic geometry should bring us nearer to the idea of a trigonometric function of a number. Should a student be permitted to draw the curve represented by the equation $y = \sin x$ with the wave-length and amplitude in any ratio that pleases him? If so, may he plot $4x^2 + y^2 = 1$ as a circle, or $x^2 + y^2 = 1$ as an ellipse? In drawing such trigonometric curves, should we not insist that the units of length on the x and y axes shall be the same, and that sin x means the sine of an angle of x radians? Queerly enough, in polar coördinates the trouble arises when we do *not* have trigonometric functions of θ rather than

[1] Read before the Mathematical Association of America, September 6, 1918.

[2] Many of the text-books state explicitly the convention that the radian is to be assumed when no other unit is indicated.

[3] Such tables are surprisingly scarce; as are also protractors reading radians and decimal parts of radians.